Pearson International GCSE in Mathematics (Specification B) (4MB1)

Two-year Scheme of Work

For first teaching from September 2016

Contents

Introduction

This Scheme of Work is based on a five-term model over two years. It is assumed that the sixth term will be dedicated to revision.

It can be used directly as a Scheme of Work for the International GCSE Mathematics (Specification B) (4MB1).

The Scheme of Work is broken up into units, so that there is greater flexibility for moving topics around to meet planning needs.

Each unit contains:

- Contents, referenced back to the specification
- Prior knowledge
- Keywords.

Each sub-unit contains:

- Recommended teaching time, although this is adaptable according to individual teaching needs
- Objectives for students at the end of the sub-unit
- Possible success criteria for students at the end of the sub-unit
- Opportunities for reasoning/problem solving
- Common misconceptions
- Notes for general mathematical teaching points.

Teachers should be aware that the estimated teaching hours are approximate and should be used as a guideline only. This scheme of work is based on 45 minute teaching lessons.

International GCSE Mathematics

(Specification B)

(4MB1)

Higher Tier Only

Scheme of Work

Using this scheme of work

The units in this scheme of work are arranged by content area, and therefore do not provide in themselves an order for how the units could be delivered. Teachers will have their own preferences for how they order the content, and the scheme of work is provided as an editable Word document to enable easy reordering of the units. Possible orders for the units at each tier are given below.

Higher tier
1. Decimals 2. Special numbers and powers 9. Algebraic manipulation 10. Expressions, formulae and rearranging formulae 11. Linear equations and inequalities 12. Sequences 30. Graphical representation of data 31. Statistical measures 3. Fractions 4. Percentages 5. Ratio and proportion 6. Indices and standard form 20. Matrices 21. Geometry of shapes 13. Distance, speed, time graphs 14. Linear graphs 15. Quadratic equations and graphs 22. Constructions and bearings 23. Perimeter, area and volume 24. Pythagoras' theorem and trigonometry 25. Transformations 16. Harder graphs 17. Simultaneous equations 32. Probability 7. Degree of accuracy 8. Set language, notation and Venn diagrams 26. Circle properties 27. Advanced trigonometry 28. Similar and congruent shapes 18. Function notation 29. Vectors 19. Calculus

Higher tier

Unit number		Title	Estimated teaching hours
Number	1	Decimals	4
	2	Special numbers, powers and roots	6
	3	Fractions	4
	4	Percentages	5
	5	Ratio and proportion	3
	6	Indices and standard form	4
	7	Degree of accuracy	4
	8	Set language, notation and Venn diagrams	6
Algebra	9	Algebraic manipulation	9
	10	Expressions, formulae and rearranging formulae	6
	11	Linear equations and inequalities	4
	12	Sequences	4
	13	Distance, speed, time graphs	2
	14	Linear graphs	7
	15	Quadratic equations and graphs	8
	16	Harder graphs	5
	17	Simultaneous equations	5
	18	Function notation	7
	19	Calculus	8
Space, shape and measure	20	Matrices	6
	21	Geometry of shapes	6
	22	Constructions and bearings	4
	23	Perimeter, area and volume	8
	24	Pythagoras' theorem and trigonometry	8
	25	Transformations	7
	26	Circle properties	6
	27	Advanced trigonometry	8
	28	Similar and congruent shapes	8
	29	Vectors	6
Handling data	30	Graphical representation of data	3
	31	Statistical measures	3
	32	Probability	6
		Total	**180**

Number : Units 1 – 9

OBJECTIVES / SPECIFICATION REFERENCES

Unit and title		Spec		Est teaching hours
1	Decimals	1H	Fractions, decimals, ratio, proportion and percentage e.g. convert recurring decimals into fractions	4
		1 I	Express numbers to a given degree of accuracy	
			e.g. use estimation to evaluate approximations to numerical calculations	
			e.g. use a scientific electronic calculator to determine numerical results	
2	Special numbers and powers	1B	Prime numbers, factors, multiples	6
			find highest common factors (HCF) and lowest common multiples (LCM)	
		1 D	Simple manipulation of surds	
		1 E	Manipulate surds, including rationalising a denominator	
		1 C	Use index notation and index laws for multiplication and division involving integer, fractional and negative powers	
		1 F	Natural numbers, integers and rational and irrational numbers	
3	Fractions	1 H	e.g. order fractions and calculate a given fraction of a given quantity	4
			e.g. express a given number as a fraction of another number	
			e.g. convert a fraction to a decimal or percentage	
		1 A	e.g. use common denominators to add and subtract fractions and mixed numbers	
			e.g. understand and use fractions as multiplicative inverses	
			e.g. multiply and divide fractions and mixed numbers	
4	Percentages	1 H	e.g. express a given number as a percentage of another number	5
			e.g. express a percentage as a fraction and as a decimal	
			e.g. understand the multiplicative nature of percentages as operators	
			e.g. solve simple percentage problems, including percentage increase and decrease	
			e.g. use reverse percentages	
			e.g. use repeated percentage change	

5	Ratio and proportion	**1 H**	e.g. use ratio notation, including reduction to its simplest form and its various links to fraction notation	3
			e.g. divide a quantity in a given ratio or ratios	
			e.g. use the process of proportionality to evaluate unknown quantities	
			e.g. calculate an unknown quantity from quantities that vary in direct proportion	
			e.g. solve word problems about ratio and proportion	
		1 G	Weights, measures and money; use and apply number in everyday personal, domestic or community life	
			Carry out calculations using standard units of mass, length, area, volume and capacity	
			Understand and carry out calculations using time, and carry out calculations using money, including converting between currencies	
6	Indices and standard form	**1 C**	Use index notation and index laws for multiplication and division of positive and negative integer powers including zero	4
		1 K	calculate with and interpret numbers in the form $a \times 10^n$ where n is an integer and $1 \leqslant a < 10$	
			Solve problems involving standard form	
7	Degree of accuracy	**1 J**	e.g. identify upper and lower bounds where values are given to a degree of accuracy	4
			Solve problems using upper and lower bounds where values are given to a degree of accuracy	
8	Set language, notation and Venn diagrams	**2 A**	The idea of a set	6
		2 C	Union and intersection of sets e.g. use the set notation $\cup$, $\cap$ and $\in$ and $\notin$	
		2 G	Understand the concept of the universal set and the empty set and the symbols for these sets	
		2 E	Complementary sets	
		2 H	Venn diagrams and their use in simple logical problems	
		2 F	Understand and use subsets	
		2 I	Use of symbols to represent sets	
		2 D	Use the notation n(A) for the number of elements in the set A	
		2 B	Use sets in abstract or practical situations	

Algebra : Units 9 – 19

OBJECTIVES / SPECIFICATION REFERENCES

Unit and title		Spec	Specification Reference	Est teaching hours
9	Algebraic manipulation	3 A	The basic processes of algebra e.g. use index notation involving fractional, negative and zero powers Take out common factors Expand the product of two or more linear expressions	9
		3 C	The factorisation of simple algebraic expressions	
		3 F	The manipulation of simple algebraic fractions, the denominators being numerical, linear or quadratic	
		3 G	Complete the square for a given quadratic expression	
		3 D	Use of the factor theorem	
		3 E	Algebraic division of a cubic by a linear factor	
		3 B	The construction, interpretation and use of formulae and their manipulation	
10	Expressions, formulae and rearranging formulae	3B	The construction, interpretation and use of formulae and their manipulation To include change of subject of a formula and substitution	6
		4 G	Variation, direct and indirect proportion	
11	Linear equations and inequalities	3G	Set up and solve linear equations	4
		3 J	Solution of linear inequalities, and the representations of solutions on the number line	
12	Sequences	3 L	The idea of a sequence Recognise sequences with a common difference or common integer sequences, and continue a given sequence	4
13	Distance, speed, time graphs	4 N	Drawing and interpretation of distance-time and speed-time graphs, and other graphs of a similar nature	2
14	Linear graphs	4 H	Rectangular Cartesian coordinates	7
		4 H	Recognise that equations of the form $y = mx + c$ are straight line graphs with gradient m and intercept on the y-axis at the point $(0, c)$ e.g. calculate the gradient of a straight line given the coordinates of two points e.g. find the equation of a straight line parallel to a given line	

		4 J	Generate points and plot graphs of linear functions	
		3 J	Solution of linear inequalities, and the representations of solutions … in two-dimensional space	
15	Quadratic equations, inequalities and graphs	**3 G**	Solution of quadratics to include solution by factorisation, by graph, by completing the square or by formula e.g. form and solve quadratic equations from data given in a context	8
		4 J	e.g. draw and interpret quadratic graphs	
		3 K	Solve quadratic inequalities in one unknown and represent the solution set on a number line	
16	Harder graphs	**4 J**	Graphs and graphical treatment of the equation: $y = Ax^3 + Bx^2 + Cx + D + \frac{E}{x} + \frac{F}{x^2}$ in which the constants are numerical and at least three of them are zero	5
		4 K	The gradients of graphs above by drawing	
		4 J	Use of the intersection of two curves (graphs) to solve equations	
17	Simultaneous equations	**3 H**	Solution of linear simultaneous equations in two unknowns e.g. interpret the equations as lines and the common solution as the point of intersection	5
		3 I	Solve simultaneous equations in two unknowns, one equation being linear and the other being quadratic	
18	Function notation	**4 A**	The idea of a function of a variable	7
		4 B	Function as a mapping or as a correspondence between the elements of two sets	
		4 C	Use functional notations of the form $\mathrm{f}(x) = \ldots$ and $\mathrm{f}: x \mapsto \ldots$	
		4 D	Domain and range of a function e.g. identify which values may need to be excluded from a domain	
		4 E	Composite functions	
		4 F	Inverse functions	
19	Calculus	**4 L**	Understand the concept of a variable rate of change	8
		4 M	Determine gradients, rates of change, stationary points, turning points (maxima and minima) by differentiation and relate these to graphs	
		4 N	Apply calculus to linear kinematics and to other simple practical problems e.g. understand the relationship between displacement or distance, velocity and speed, and acceleration, for example: $\frac{\mathrm{d}s}{\mathrm{d}t} = v$ and $\frac{\mathrm{d}v}{\mathrm{d}t} = a$	

Shape, space and measure : Units 20 – 29

OBJECTIVES / SPECIFICATION REFERENCES

Unit and title		Specification Reference		Est teaching hours
		Spec		
20	Matrices	**5 A**	Representation of data by a matrix	6
		5 B	Addition and multiplication of matrices	
		5 C	Multiplication of a matrix by a scalar	
		5 D	Unit (identity) matrix and zero (null) matrix	
		5 E	Determinants and inverses of non-singular 2×2 matrices	
		5 F	Transformations of the plane associated with 2×2 matrices	
21	Geometry of shapes	**6 A**	Geometrical properties of Euclidean space	6
		6 B	Geometrical reasoning	
		6 C	Angle properties of parallel lines, triangles and polygons, including regular polygons	
		6 D	Properties of the parallelogram, rectangle, square, rhombus, trapezium and kite	
		6 E	Symmetry about a point, line or plane e.g. complete shapes with a given axis of symmetry and order of rotational symmetry	
22	Constructions and bearings	**6 M**	Loci in two dimensions	4
		6 N	Constructions of bisector of an angle and of perpendicular bisector (mediator) of a straight line [Constructions using only ruler and compasses]	
		9 D	Bearings e.g. understand three-figure bearings	
23	Perimeter, area and volume	**7 A**	Length, area, and volume	8
		7 B	Mensuration of two–dimensional shapes, rectangle, parallelogram, trapezium, triangle, circle	
		7 C	Mensuration of the three–dimensional shapes, right circular cylinder, right circular cone and sphere, cuboid, pyramid, prism	
		7 D	Length of an arc, area of a sector of a circle	
24	Pythagoras' theorem and trigonometry	**6 F**	Know, understand and use Pythagoras' theorem in two dimensions	8
		9 A	Use of sine, cosine and tangent of angles up to 180°	
		9 B	Solution of problems in two dimensions by calculation and by drawing	
		9 C	Angles of elevation and depression	
25	Transformations	**8 J**	Transformations of the plane i.e. Reflections in any line, rotations about any point , translations , enlargements	7
		8 K	Combination of transformations	

		8 L	Multiplication of a vector by a matrix e.g. to include the finding of a matrix for a given transformation of the plane, using $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$	
26	Circle properties	**6 L**	Properties of a cyclic quadrilateral	6
		6 K	Chord, angle and tangent properties of circles. To include knowledge of the intersecting chord properties (both internal and external) and the alternate segment theorem	
27	Advanced trigonometry	**6 F**	Use Pythagoras' theorem in three dimensions	8
		9 B	Solution of problems in three dimensions by calculation and by drawing e.g. use of the sine and cosine rule Area of a triangle $= \frac{1}{2}ab\sin C$	
28	Similar and congruent shapes	**6 G**	Similarity: areas and volumes of similar figures	8
		6 H	Prove the similarity of two triangles	
		6 I	Congruent shapes	
		6 J	Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles	
29	Vectors	**8 A**	Scalar and vector quantities	6
		8 B	Understand and use vector notation including column vectors	
		8 C	Representation of a vector by a directed line segment	
		8 D	Parallel vectors, unit vectors and position vectors	
		8 E	Sum and difference of two vectors	
		8 F	Modulus (magnitude) of a vector	
		8 G	Multiplication of a vector by a scalar	
		8 H	Find the resultant of two or more vectors	
		8 I	Apply vector methods for simple geometrical proofs	

Handling Data : Units 30 – 32

OBJECTIVES / SPECIFICATION REFERENCES

Unit and title		Specification Reference		Est teaching hours
		Spec		
30	Graphical representation of data	**10 A**	Graphical representation of numerical data e.g. bar charts, pie charts and histograms but not cumulative frequency graphs	3
31	Statistical measures	**10 B**	Determination of the mean, median, mode and range for a discrete data set	3
		10 C	Calculation of an estimate of the mean of a larger number of quantities given in grouped frequencies	
		10 D	Determination of a modal class and the class containing the median for grouped data	
32	Probability	**10 E**	Understand the language and basic concepts of probability e.g. probability scale, sample spaces, relative frequency, probabilities and complements	6
		10 F	Use of addition rule for two or more mutually exclusive events	
		10 G	Use of product rule for two or more independent events	
		10 H	Determination of the probability of two or more independent events including the use tree diagrams	
		10 I	Using simple conditional probability for combined events	
		10 J	Finding very simple conditional probability	
		10 K	Understand and use the term ‘expected frequency’	

1. Decimals

Teaching time
3-5 hours

OBJECTIVES

1 H	Fractions, decimals, ratio, proportion and percentage e.g. convert recurring decimals into fractions
1 I	Express numbers to a given degree of accuracy
	e.g. use estimation to evaluate approximations to numerical calculations
	e.g. use a scientific electronic calculator to determine numerical results

POSSIBLE SUCCESS CRITERIA

Estimate the value of $\frac{36.5+8.34}{0.154}$ and evaluate the expression on your calculator.

Give your answer correct to 3 significant figures.

Change $0.4\dot{5}$ into a fraction in its simplest form.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Use of decimals within a problem.

Show algebraically that $3.0\dot{1}$ can be written as $3\frac{1}{90}$

Links with other areas of mathematics can be made by using surds in Pythagoras and when using trigonometric ratios.

COMMON MISCONCEPTIONS

Significant figure and decimal place rounding are often confused.
Some students may think 35 934 = 36 to two significant figures.

NOTES

The expectation is that much of this work will be reinforced throughout the course.
Make sure students are absolutely clear about the difference between significant figures and decimal places.

EXAMPLE QUESTIONS FROM SAMs: -

(None)

2. Special numbers and powers	Teaching time 5-7 hours

OBJECTIVES

1 B	Prime numbers, factors, multiples find highest common factors (HCF) and lowest common multiples (LCM)
1 D	Simple manipulation of surds
1 E	manipulate surds, including rationalising a denominator
1 C	use index notation and index laws for multiplication and division involving integer, fractional and negative powers
1 F	Natural numbers, integers and rational and irrational numbers

POSSIBLE SUCCESS CRITERIA

What is the value of 2^5?
Find the HCF and LCM of 12 and 20
Write a number as a product of its prime factors.
Prove that the square root of 45 lies between 6 and 7
Simplify $\sqrt{40}$

Rationalise the denominator of $\dfrac{5}{\sqrt{10}}$; $\dfrac{6}{1+\sqrt{2}}$

Identify the irrational numbers from a list, e.g. $\sqrt{6\frac{1}{4}}$ $\dfrac{1}{\pi}$ $\dfrac{5}{19}$ $\dfrac{\sqrt{6}}{2\sqrt{3}}$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that use indices instead of integers will provide rich opportunities to apply the knowledge in this unit in other areas of mathematics.

COMMON MISCONCEPTIONS

The order of operations is often not applied correctly when squaring negative numbers, and many calculators will reinforce this misconception.

NOTES

Students need to know how to enter negative numbers into their calculator.
Use negative number and not minus number to avoid confusion with calculations.
Students need to be encouraged to learn squares from 2 × 2 to 15 × 15 and cubes of 2, 3, 4, 5 and 10, and corresponding square and cube roots.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 4 and Q 10

3. Fractions

Teaching time
3-5 hours

OBJECTIVES

1 H	e.g. order fractions and calculate a given fraction of a given quantity
	e.g. express a given number as a fraction of another number
	e.g. convert a fraction to a decimal or percentage
1 A	e.g. use common denominators to add and subtract fractions and mixed numbers
	e.g. understand and use fractions as multiplicative inverses
	e.g. multiply and divide fractions and mixed numbers

POSSIBLE SUCCESS CRITERIA

Express a given number as a fraction of another, including where the fraction is, for example, greater than 1, e.g. $\frac{120}{100} = 1\frac{2}{10} = 1\frac{1}{5}$

Answer the following: James delivers 56 newspapers. $\frac{3}{8}$ of the newspapers have a magazine.

How many of the newspapers have a magazine?
Prove whether a fraction is terminating or recurring.
Convert a fraction to a decimal including where the fraction is greater than 1

Convince me that 0.125 is $\frac{1}{8}$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Many of these topics provide opportunities for reasoning in real-life contexts, particularly percentages.

Calculate original values and evaluate statements in relation to this value justifying which statement is correct.

COMMON MISCONCEPTIONS

The larger the denominator, the larger the fraction.

Incorrect links between fractions and decimals, such as thinking that $\frac{1}{5}$ = 0.15, 5% = 0.5, 4% = 0.4, etc.

NOTES

Ensure that you include fractions where only one of the denominators needs to be changed, in addition to where both need to be changed for addition and subtraction.
Include multiplying and dividing integers by fractions.
Encourage use of the fraction button.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 2

4. Percentages

Teaching time
4-6 hours

OBJECTIVES

1H	e.g. express a given number as a percentage of another number;
	e.g. express a percentage as a fraction and as a decimal
	e.g. understand the multiplicative nature of percentages as operators
	e.g. solve simple percentage problems, including percentage increase and decrease
	e.g. use reverse percentages
	e.g. use repeated percentage change

POSSIBLE SUCCESS CRITERIA

Be able to work out the price of a deposit, given the price of a sofa is £480 and the deposit is 15% of the price, without a calculator.
Find fractional percentages of amounts, with and without using a calculator.
Work out 56 cm as a percentage of 2.5 m.
Find the original price when the sale price of an item is £68 following a reduction of 15%

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Many of these topics provide opportunities for reasoning in real-life contexts, particularly percentages.

Calculate original values and evaluate statements in relation to this value justifying which statement is correct.

COMMON MISCONCEPTIONS

Incorrect links between fractions and decimals, such as thinking that $\frac{1}{5}$ = 0.15, 5% = 0.5, 4% = 0.4, etc.
It is not possible to have a percentage greater than 100%.

NOTES

Students should be reminded of basic percentages.
Amounts of money should always be rounded to the nearest penny, except where successive calculations are done (i.e. repeated percentage change, which is covered in a later unit).
Emphasise the use of percentages in real-life situations.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 1 and 02 Q 1

5. Ratio and proportion

Teaching time
2-4 hours

OBJECTIVES

1 H	e.g. use ratio notation, including reduction to its simplest form and its various links to fraction notation
	e.g. divide a quantity in a given ratio or ratios
	e.g. use the process of proportionality to evaluate unknown quantities
	e.g. calculate an unknown quantity from quantities that vary in direct proportion
	e.g. solve word problems about ratio and proportion
1G	Weights, measures and money use and apply number in everyday, personal, domestic or community life
	carry out calculations using standard units of mass, length, area, volume and capacity
	understand and carry out calculations using time, and carry out calculations using money, including converting between currencies

POSSIBLE SUCCESS CRITERIA

Write/interpret a ratio to describe a situation such as 1 blue for every 2 red …, 3 adults for every 10 children …
Recognise that two paints mixed red to yellow 5 : 4 and 20 : 16 are the same colour.
When a quantity is split in the ratio 3:5, what fraction does each person get?
Find amounts for three people when amount for one given.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems involving sharing in a ratio that include percentages rather than specific numbers such can provide links with other areas of mathematics.

> In a youth club the ratio of the number of boys to the number of girls is 3 : 2 . 30% of the boys are under the age of 14 and 60% of the girls are under the age of 14. What percentage of the youth club is under the age of 14?

COMMON MISCONCEPTIONS

Students often identify a ratio-style problem and then divide by the number given in the question, without fully understanding the question.

NOTES

Three-part ratios are usually difficult for students to understand.
Also include using decimals to find quantities.
Use a variety of measures in ratio and proportion problems.

EXAMPLE QUESTIONS FROM SAMs: (None)

6. Indices and standard form	Teaching time 3-5 hours

OBJECTIVES

1 C	use index notation and index laws for multiplication and division of positive and negative integer powers including zero
1 K	calculate with and interpret numbers in the form $a \times 10^n$ where n is an integer and $1 \leqslant a < 10$
	solve problems involving standard form

POSSIBLE SUCCESS CRITERIA

Evaluate $(2^3 \times 2^5) \div 2^4$, 4^0, $8^{-\frac{2}{3}}$

Work out the value of n in $40 = 5 \times 2^n$

Write 51080 in standard form.
Write 3.74×10^{-6} as an ordinary number.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements and justify which answer is correct by providing a counterargument by way of a correct solution.

COMMON MISCONCEPTIONS

Some students may think that any number multiplied by a power of 10 qualifies as a number written in standard form.

NOTES

Standard form is used in science and there are lots of cross-curricular opportunities.
Students need to be provided with plenty of practice in using standard form with calculators.

EXAMPLE QUESTIONS FROM SAMs: (None)

7. Degree of accuracy	Teaching time 3-5 hours

OBJECTIVES

1 J	e.g. identify upper and lower bounds where values are given to a degree of accuracy
	solve problems using upper and lower bounds where values are given to a degree of accuracy

POSSIBLE SUCCESS CRITERIA

Round 16,000 people to the nearest 1000
Round 1100 g to 1 significant figure.
Work out the upper and lower bounds of a formula where all terms are given to 1 decimal place.
Be able to justify that measurements to the nearest whole unit may be inaccurate by up to one half in either direction.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

This sub-unit provides many opportunities for students to evaluate their answers and provide counterarguments in mathematical and real-life contexts, in addition to requiring them to understand the implications of rounding their answers.

COMMON MISCONCEPTIONS

Students readily accept the rounding for lower bounds, but take some convincing in relation to upper bounds.

NOTES

Students should use ‘half a unit above’ and ‘half a unit below’ to find upper and lower bounds.
Encourage use of a number line when introducing the concept.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 8

8. Set language, notation and Venn diagrams

Teaching time
5-7 hours

OBJECTIVES

2 A	The idea of a set
2 C	Union and intersection of sets e.g. use the set notation $\cup$, $\cap$ and $\in$ and $\notin$
2 G	understand the concept of the universal set and the empty set and the symbols for these sets
2 E	Complementary sets
2 H	Venn diagrams and their use in simple logical problems
2 F	understand and use subsets
2 I	use of symbols to represent sets
2 D	use the notation n(A) for the number of elements in the set A
2 B	use sets in abstract or practical situations

POSSIBLE SUCCESS CRITERIA

Universal set is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {1, 2, 3, 4, 5, 6}, B = { 2, 4, 6, 8}; Write down $A \cap B$, $A \cup B$
C = {1, 3, 5}; write down C'
Is 4 $\in C$, is 4 $\in A$, is C a subset of A?
Find n(A).
Draw a Venn diagram to show the universal set, A, B and C
If a number is picked at random, find P($A \cap B$)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Given Universal set is {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {5, 7, 9} and B = {1, 3, 5, 7}
Write down a possible set C so that $A \cap C$ = {7} and C has 4 members.

COMMON MISCONCEPTIONS

A = {5, 7, 9} and B = {1, 3, 5, 7} then $A \cup B$ = {1, 3, 5, 5, 7, 7, 9}

NOTES

When drawing a Venn diagram it is a good idea to put members in the intersection first.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 11 and 02 Q 3

9. Algebraic manipulation

Teaching time
8-10 hours

OBJECTIVES

3 A	The basic processes of algebra
	e.g. use index notation involving fractional, negative and zero powers
	take out common factors
	expand the product of two or more linear expressions
3 C	The factorisation of simple algebraic expressions
3 F	The manipulation of simple algebraic fractions, the denominators being numerical, linear or quadratic
3 G	complete the square for a given quadratic expression
3 D	Use of the factor theorem
3 E	Algebraic division of a cubic by a linear factor
3 B	The construction, interpretation and use of formulae and their manipulation

POSSIBLE SUCCESS CRITERIA

Simplify $4p - 2q^2 + 1 - 3p + 5q^2$

Simplify $z^4 \times z^3$, $y^3 \div y^2$, $(a^7)^2$, $\left(8x^6y^4\right)^{\frac{1}{3}}$ Factorise $15x^2y - 35x^2y^2$; $6x^2 - 7x + 1$

Expand and simplify $3(t - 1) + 57$; $(3x + 2)(4x - 1)$; $(x + 7)(x - 1)(2x + 1)$
Use the factor theorem to show that $(2x - 1)$ is a factor of $2x^3 - 5x^2 - 4x + 3$
Hence solve $2x^3 - 5x^2 - 4x + 3 = 0$
Use fractions when working in algebraic situations.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements and justify which answer is correct by providing a counterargument by way of a correct solution.

COMMON MISCONCEPTIONS

When expanding two linear expressions, poor number skills involving negatives and times tables will become evident.

NOTES

Some of this will be a reminder from Key Stage 3 and could be introduced through investigative material such as handshake, frogs etc.
Students will be asked to show ‘algebraic working’ when solving equations. Solutions with no working will score no marks.
Students can leave their answer in fraction form where appropriate. Emphasise that fractions are more accurate in calculations than rounded percentage or decimal equivalents.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 21, Q 26 and 02 Q 2

10. Expressions, formulae and rearranging equations

Teaching time
5-7 hours

OBJECTIVES

3 B	The construction, interpretation and use of formulae and their manipulation
	To include change of subject of a formula and substitution
4 G	Variation, direct and indirect proportion

POSSIBLE SUCCESS CRITERIA

Find the value of $3x^2 - 2x$ for different values of x.
Find the value a in $v^2 = u^2 + 2as$ given values of the other variables.
Make a the subject of $v^2 = u^2 + 2as$
Make y the subject of $t = \sqrt{\frac{2-3y}{4}}$
Make t the subject of $a = \frac{2t+b}{3-t}$
Given that y is inversely proportional to x^2 ,and that when $x = 2$, $y = 3$, find a formula for y in terms of x.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Justify and infer relationships in real-life scenarios to direct and inverse proportion such as ice cream sales and sunshine.

COMMON MISCONCEPTIONS

Confusing direct and inverse proportion.

NOTES

Students should be reminded to show all stages in their working.
Consider using science contexts for problems involving inverse proportionality, e.g. volume of gas inversely proportional to the pressure or frequency is inversely proportional to wavelength.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 19

11. Linear equations and inequalities

Teaching time
5-7 hours

OBJECTIVES

3G	set up and solve linear equations
3 J	Solution of linear inequalities, and the representations of solutions on the number line

POSSIBLE SUCCESS CRITERIA

Solve $5(x + 3) = 2x – 7$
Use inequality symbols to compare numbers.
Given a list of numbers, represent them on a number line using the correct notation.
Solve equations involving inequalities.
Solve $4x + 5 > x + 1$
Solve $\frac{2x-1}{3}-\frac{x+1}{2}=5$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require student to justify why certain values in a solution can be ignored.
Set up and solve problems involving linear equations.

COMMON MISCONCEPTIONS

When solving inequalities students often state their final answer as a number quantity, and exclude the inequality or change it to =
Some students believe that –6 is greater than –3
When solving equations like $\frac{2x-1}{3}-\frac{x+1}{2}=5$ the common error is to forget to use the negative sign when expanding brackets.

NOTES

Emphasise the importance of leaving their answer as an inequality (and not changing it to =).
Students can leave their answers in fractional form where appropriate.
Ensure that correct language is used to avoid reinforcing misconceptions: for example, 0.15 should never be read as ‘zero point fifteen’, and 5 > 3 should be read as ‘five is greater than 3’, not ‘5 is bigger than 3’

EXAMPLE QUESTIONS FROM SAMs: 01 Q 3

12. Sequences

Teaching time
3-5 hours

OBJECTIVES

3 L	The idea of a sequence
	recognise sequences with a common difference or common integer sequences, and continue a given sequence

POSSIBLE SUCCESS CRITERIA

Given a sequence, 'which is the 1st term greater than 50?'
Continue the sequence 3, 8, 15, 24 ...
Given the sequence 12, 7, 2, -3… find an expression in terms of n for the nth term.
Be able to solve problems involving sequences from real-life situations, such as:

- What is the amount of money after x months saving the same amount, or the height of a tree that grows 6 m per year?

Given the sequence 5, 8, 11, 14… find the 50th term.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements about whether or not specific numbers or patterns are in a sequence and justify the reasons.

COMMON MISCONCEPTIONS

Students struggle to relate the position of the term to "n".
Writing $n + 3$ instead of $3n - 1$ for the nth term of 2, 5, 8, 11…

NOTES

Emphasise use of $3n$ meaning 3 x n.
Students need to be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the nth term.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 5

13. Distance, speed, time graphs

Teaching time
1-3 hours

OBJECTIVES

4 N	drawing and interpretation of distance/time and speed/time graphs, and other graphs of a similar nature

POSSIBLE SUCCESS CRITERIA

Interpret a description of a journey into a distance–time or speed–time graph.
Calculate various measures given a graph.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Speed/distance graphs can provide opportunities for interpreting non-mathematical problems as a sequence of mathematical processes, whilst also requiring students to justify their reasons why one vehicle is faster than another.

COMMON MISCONCEPTIONS

Reading scales incorrectly is a common cause of errors.

NOTES

Careful annotation should be encouraged: it is good practice to label the axes and check that students understand the scales.
Use various measures in the distance–time and velocity–time graphs, including miles, kilometres, seconds, and hours, and include large numbers in standard form.
Ensure that you include axes with negative values to represent, for example, time before present time, temperature or depth below sea level.

EXAMPLE QUESTIONS FROM SAMs: - 02 Q 6

14. Linear graphs

Teaching time
6-8 hours

OBJECTIVES

4 H	recognise that equations of the form $y = mx + c$ are straight line graphs with gradient m and intercept on the y-axis at the point $(0, c)$
	e.g. calculate the gradient of a straight line given the coordinates of two points
	e.g. find the equation of a straight line parallel to a given line
4 J	generate points and plot graphs of linear functions
3 J	Solution of linear inequalities, and the representations of solutions in two-dimensional space

POSSIBLE SUCCESS CRITERIA

Find the equation of the line passing through two coordinates by calculating the gradient first.
Understand that the form $y = mx + c$ or $ax + by = c$ represents a straight line.
Show the region defined by $x < 3$, $y > 1$, $y < 3x + 2$
Find an equation of the line that goes through (1, 2) and is parallel to $3y + 2x = 5$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Given an equation of a line, provide a counterargument as to whether or not another equation of a line is parallel to the first line.
Decide if lines are parallel without drawing them and provide reasons.

COMMON MISCONCEPTIONS

Students can find visualisation of a question difficult, especially when dealing with gradients resulting from negative coordinates.

NOTES

Encourage students to sketch what information they are given in a question – emphasise that it is a sketch.
Careful annotation should be encouraged – it is good practice to label the axes and check that students understand the scales.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 15

15. Quadratic equations, inequalities and graphs	Teaching time 7-9 hours

OBJECTIVES

3 G	Solution of quadratics to include solution by factorisation, by graph, by completing the square or by formula
	e.g. form and solve quadratic equations from data given in a context
4 J	e.g. draw and interpret quadratic graphs
3 K	solve quadratic inequalities in one unknown and represent the solution set on a number line

POSSIBLE SUCCESS CRITERIA

Solve $3x^2 + 4 = 112$
Solve $2x^2 + 3x + 1 = 0$
Draw the graph of $y = x^2 + 5x + 6$
Know that the quadratic formula can be used to solve all quadratic equations.
Select the more efficient method from formula, factorising or completing the square.
Have an understanding of solutions that can be written in surd form.
Solve $x^2 < 9$; $2x^2 + 3x + 1 < 0$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require students to set up and solve a quadratic equation or inequality.

COMMON MISCONCEPTIONS

Using the formula involving negatives can result in incorrect answers.
All workings must be shown when solving quadratic equations, including substitution into the quadratic formula.

NOTES

Remind students to use brackets for negative numbers when using a calculator, and remind them of the importance of knowing when to leave answers in surd form.
Reinforce the fact that some problems may produce one inappropriate solution, which can be ignored.
Clear presentation of working out is essential.
Link with graphical representations.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 27

16. Harder graphs and transformation of graphs

Teaching time
4-6 hours

OBJECTIVES

4 J	Graphs and graphical treatment of the equation: $$y = Ax^3 + Bx^2 + Cx + D + \frac{E}{x} + \frac{F}{x^2}$$ in which the constants are numerical and at least three of them are zero
4 K	The gradients of graphs above by drawing
4 J	Use of the intersection of two curves (graphs) to solve equations

POSSIBLE SUCCESS CRITERIA

Select and use the correct mathematical techniques to draw graphs.
Identify a variety of functions by the shape of the graph.
Find the gradient, at a point, of a non-linear graph.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Match equations of quadratics, cubics, reciprocal functions with their graphs by recognising the shape or by sketching.

COMMON MISCONCEPTIONS

Students struggle with the concept of solutions and what they represent in concrete terms.

NOTES

Use lots of practical examples to help model the quadratic function, e.g. draw a graph to model the trajectory of a projectile and predict when/where it will land.
Ensure axes are labelled and pencils used for drawing.
Graphical calculations or appropriate ICT will allow students to see the impact of changing variables within a function.

EXAMPLE QUESTIONS FROM SAMs: 02 Q 8

17. Simultaneous equations	Teaching time 4-6 hours

OBJECTIVES

3 H	Solution of linear simultaneous equations in two unknowns
	e.g. interpret the equations as lines and the common solution as the point of intersection
3 I	solve simultaneous equations in two unknowns, one equation being linear and the other being quadratic

POSSIBLE SUCCESS CRITERIA

Solve the simultaneous equations $2x + 5y = -14$; $3x - 4y = 25$
Solve the simultaneous equations $x^2 + y^2 = 18$; $2x + 1 = y$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require students to set up and solve a pair of simultaneous equations in a real-life context, such as 2 adult tickets and 1 child ticket cost £28, and 1 adult ticket and 3 child tickets cost £34. How much does 1 adult ticket cost?
Link the solution of simultaneous equations to their graphical representation.

COMMON MISCONCEPTIONS

Some students always discard solutions with negative values.

NOTES

Reinforce the fact that some problems may produce one inappropriate solution, which can be ignored.
Clear presentation of working out is essential.
Link with graphical representations.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 18 and 02 Q 5

18. Function notation	Teaching time 6-8 hours

OBJECTIVES

4 A	The idea of a function of a variable
4 B	Function as a mapping or as a correspondence between the elements of two sets
4 C	use functional notations of the form $f(x) = \ldots$ and $f: x \mapsto \ldots$
4 D	Domain and range of a function e.g. identify which values may need to be excluded from a domain
4 E	Composite functions
4 F	Inverse functions

POSSIBLE SUCCESS CRITERIA

Given f(x) = 3 – 5x; find f(2), $f^{-1}(3)$

Given g(x) = $\dfrac{2}{3-x}$, write down the value of x that must be omitted from any domain of g.

Find fg(4); gf(4)

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Forming and solving equations using functions. E.g. solve f(x) = g(x)
Give the graph of f(x) and use that to find f(3) and f(x) = 2

COMMON MISCONCEPTIONS

Confusing gf(x) with fg(x)

NOTES

Link with algebraic manipulation and equation solving.

EXAMPLE QUESTIONS FROM SAMs: none

19. Calculus	Teaching time 7-9 hours

OBJECTIVES

4 L	understand the concept of a variable rate of change
4 M	determine gradients, rates of change, stationary points, turning points (maxima and minima) by differentiation and relate these to graphs
4 N	apply calculus to linear kinematics and to other simple practical problems e.g. understand the relationship between displacement or distance, velocity and speed, and acceleration, for example: $\frac{\mathrm{d}s}{\mathrm{d}t} = v$ and $\frac{\mathrm{d}v}{\mathrm{d}t} = a$

POSSIBLE SUCCESS CRITERIA

Differentiate $8x^3 + 3x + 2$; $\frac{2}{x^2} + 3x$

Find the turning point of $y = x^2 + 8x - 20$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Find the values of x for which the graph of $y = x^2 - x + 3$ has a gradient of 7
Given that $s = t^3 - 2t^2$ find the value of t for which the particle is instantaneously at rest.

COMMON MISCONCEPTIONS

3 differentiates to 3 (rather than 0)

NOTES

Link with solving linear and quadratic equations.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 25 and 02 Q 4

20. Matrices

Teaching time
5-7 hours

OBJECTIVES

5 A	Representation of data by a matrix
5 B	Addition and multiplication of matrices
5 C	Multiplication of a matrix by a scalar
5 D	Unit (identity) matrix and zero (null) matrix
5 E	Determinants and inverses of non-singular 2×2 matrices
5 F	Transformations of the plane associated with 2×2 matrices

POSSIBLE SUCCESS CRITERIA

Given the matrices **A** = $\begin{pmatrix} -1 & 3 \end{pmatrix}$ and **B** = $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ find **AB** and **BA**

Transform a given triangle with the matrix **L** = $\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$

Describe the transformation represented by **L**

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Given the matrix **M** = $\begin{pmatrix} -1 & -2 \\ 3 & 1 \end{pmatrix}$, find the matrix **N** such that **NM** = $\begin{pmatrix} 5 & 10 \\ 0 & 5 \end{pmatrix}$

Find the matrix representing the transformations of a reflection in the line $y = x$ followed by a rotation of 90° about the origin.

COMMON MISCONCEPTIONS

AB = BA

Thinking that transformation **LM** means the transformation represented by **L** followed by the transformation represented by **M**

NOTES

Links with simultaneous equations and use of inverse matrix to solve simultaneous linear equations.
Some questions may lead to other algebraic equations.
Links with work on transformations.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 16 and 02 Q 10

21. Geometry of shapes

Teaching time
5-7 hours

OBJECTIVES

6 A	Geometrical properties of Euclidean space
6 B	Geometrical reasoning
6 C	Angle properties of parallel lines, triangles and polygons, including regular polygons
6 D	Properties of the parallelogram, rectangle, square, rhombus, trapezium and kite
6 E	Symmetry about a point, line or plane e.g. complete shapes with a given axis of symmetry and order of rotational symmetry

POSSIBLE SUCCESS CRITERIA

Name all quadrilaterals that have a specific property.
Given the size of its exterior angle, how many sides does the polygon have?
What is the same and what is different between families of polygons?
Given a geometric diagram, find the value of a given angle and give a reason for each stage of working.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step “angle chasing”-style problems that involve justifying how students have found a specific angle will provide opportunities to develop a chain of reasoning.
Geometrical problems involving algebra, whereby equations can be formed and solved, allow students the opportunity to make and use connections with different parts of mathematics.

COMMON MISCONCEPTIONS

Some students will think that all trapezia are isosceles, or a square is only square if ‘horizontal’, or a ‘non-horizontal’ square is called a diamond.
Incorrectly identifying the ‘base angles’ (i.e. the equal angles) of an isosceles triangle when not drawn horizontally.

NOTES

Students must be encouraged to use geometrical language appropriately, ‘quote’ the appropriate reasons for angle calculations and show step-by-step deduction when solving multi-step problems.
Emphasise that diagrams in examinations are seldom drawn accurately.
Use triangles to find angle sums of polygons; this could be explored algebraically as an investigation.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 6

22. Constructions and bearings

Teaching time
3-5 hours

OBJECTIVES

6 M	Loci in two dimensions
6 N	Constructions of bisector of an angle and of perpendicular bisector (mediator) of a straight line [Constructions using only ruler and compasses]
9 D	Bearings e.g. understand three-figure bearings

POSSIBLE SUCCESS CRITERIA

Able to read and construct scale drawings.
When given the bearing of a point A from point B, can work out the bearing of B from A.
Know that scale diagrams, including bearings and maps, are 'similar' to the real-life examples.
Construct the perpendicular bisector of a given angle.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems involving combinations of bearings and scale drawing can provide a rich opportunity to link with other areas of mathematics and allow students to justify their findings.

COMMON MISCONCEPTIONS

Correct use of a protractor may be an issue.

NOTES

Drawings should be done in pencil.
Construction lines should not be erased.

EXAMPLE QUESTIONS FROM SAMs: none

23. Perimeter, area and volume

Teaching time
7-9 hours

OBJECTIVES

7 A	Length, area, and volume
7 B	Mensuration of two–dimensional shapes, rectangle, parallelogram, trapezium, triangle, circle
7 C	Mensuration of the three–dimensional shapes, right circular cylinder, right circular cone and sphere, cuboid, pyramid, prism
7 D	Length of an arc, area of a sector of a circle

POSSIBLE SUCCESS CRITERIA

Calculate the area and/or perimeter of shapes with different units of measurement.
Understand that answers in terms of π are more accurate.
Calculate the perimeters and/or areas of circles and sectors of circles given the radius or diameter and vice versa.
Work out the length given the area of the cross-section and volume of a cuboid.
Given two solids with the same volume and the dimensions of one, write and solve an equation in terms of π to find the dimensions of the other.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Using compound shapes or combinations of polygons that require students to subsequently interpret their result in a real-life context.
Multi-step problems, including the requirement to form and solve equations, provide links with other areas of mathematics.
Combinations of 3D forms such as a cone and a sphere where the radius has to be calculated given the total height.

COMMON MISCONCEPTIONS

Students often get the concepts of area and perimeter confused.
Students often get the concepts of surface area and volume confused.

NOTES

Encourage students to draw a sketch where one isn't provided.
Ensure that examples use different metric units of length, including decimals.
Emphasise the need to learn the circle formulae; "Cherry Pie's Delicious" and "Apple Pies are too" are good ways to remember them.
Ensure that students know it is more accurate to leave answers in terms of π, but only when asked to do so.

EXAMPLE QUESTIONS FROM SAMs: 01 Q22 and 02 Q 11

24. Pythagoras' theorem and trigonometry

Teaching time
7-9 hours

OBJECTIVES

6 F	know, understand and use Pythagoras' theorem in two dimensions
9 A	Use of sine, cosine and tangent of angles up to 180°
9 B	Solution of problems in two dimensions by calculation and by drawing
9 C	Angles of elevation and depression

POSSIBLE SUCCESS CRITERIA

Does 2, 3, 6 give a right-angled triangle?
Justify when to use Pythagoras' theorem and when to use trigonometry.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combined triangle problems that involve consecutive application of Pythagoras' theorem or a combination of Pythagoras' theorem and the trigonometric ratios.
Link to 'real-life' situations. E.g. link with bearings and scale drawings.

COMMON MISCONCEPTIONS

Answers may be displayed on a calculator in surd form.
Students forget to square root their final answer, or round their answer prematurely.

NOTES

Students may need reminding about surds.
Scale drawings are not acceptable.
Calculators need to be in degree mode.
Use a suitable mnemonic to remember SOHCAHTOA.
Use Pythagoras' theorem and trigonometry together.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 24

25. Transformations	Teaching time 6-8 hours

OBJECTIVES

8 J	Transformations of the plane i.e. reflections in any line, rotations about any point , translations , enlargements
8 K	Combination of transformations
8 L	Multiplication of a vector by a matrix e.g. to include the finding of a matrix for a given transformation of the plane, using $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$

POSSIBLE SUCCESS CRITERIA

Understand that translations are specified by a distance and direction (using a vector).
Recognise that enlargements preserve angle but not length.
Understand that distances and angles are preserved under rotations, reflections and translations so that any shape is congruent to its image.
Understand that similar shapes are enlargements of each other and angles are preserved.
Use the vectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ to determine the transformation represented by $\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to explore the effect of reflecting in two parallel mirror lines and combining transformations.

COMMON MISCONCEPTIONS

Students often use the term ‘transformation’ when describing transformations instead of the required information.
Lines parallel to the coordinate axes often get confused.
Students often confuse the terms “translation” and “transformation”.

NOTES

Emphasise the need to describe the transformations fully, and if asked to describe a ‘single’ transformation students should not include two types.
Find the centre of rotation, by trial and error and by using tracing paper. Include centres on or inside shapes.

EXAMPLE QUESTIONS FROM SAMs: 02 Q 10

26. Circle properties

Teaching time
5-7 hours

OBJECTIVES

6 L	Properties of a cyclic quadrilateral
6 K	Chord, angle and tangent properties of circles. To include knowledge of the intersecting chord properties (both internal and external) and the alternate segment theorem

POSSIBLE SUCCESS CRITERIA

Justify clearly missing angles on diagrams using the various circle theorems, giving a reason for each stage in working.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that involve a clear chain of reasoning and provide counterarguments to statements. Can be linked to other areas of mathematics by incorporating trigonometry and Pythagoras' theorem.

COMMON MISCONCEPTIONS

Much of the confusion arises from mixing up the diameter and the radius.
There is often confusion when identifying cyclic quadrilaterals.

NOTES

Reasoning needs to be carefully constructed and correct notation should be used throughout.
Students should label any diagrams clearly, as this will assist them; particular emphasis should be made on labelling any radii in the first instance.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 20

27. Advanced trigonometry

Teaching time
7-9 hours

OBJECTIVES

6 F	use Pythagoras' theorem in three dimensions
9 B	Solution of problems in three dimensions by calculation and by drawing e.g. use of the sine and cosine rule Area of a triangle $= \frac{1}{2}ab\sin C$

POSSIBLE SUCCESS CRITERIA

Find the area of a segment of a circle given the radius and length of the chord.
Justify when to use the cosine rule, sine rule, Pythagoras' theorem or normal trigonometric ratios to solve problems.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Triangles formed in a semicircle can provide links with other areas of mathematics.
Multi-step problems requiring the use of both the sine rule and cosine rule.

COMMON MISCONCEPTIONS

Not using the correct rule, or attempting to use 'normal trig' in non-right-angled triangles.
When finding angles, students will often be unable to rearrange the cosine rule or fail to find the inverse of cos θ.

NOTES

The cosine rule is used when we have SAS and used to find the side opposite the 'included' angle or when we have SSS to find an angle.
Ensure that finding angles with 'normal trig' is refreshed prior to this topic.
Students may find it useful to be reminded of simple geometrical facts, i.e. the shortest side is always opposite the shortest angle in a triangle.
In multi-step questions emphasise the importance of not rounding prematurely and using exact values where appropriate.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 28 and 02 Q 7

28. Similar and congruent shapes

Teaching time
7-9 hours

OBJECTIVES

6 G	Similarity: areas and volumes of similar figures
6 H	prove the similarity of two triangles
6 I	Congruent shapes
6 J	understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles

POSSIBLE SUCCESS CRITERIA

Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not.
Understand that enlargement does not have the same effect on area and volume.
Given the volumes of two similar shapes and the surface area of one, find the surface area of the other shape.
Prove that two triangles are congruent using one of the 4 conditions.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step questions that require calculating missing lengths of similar shapes prior to calculating area of the shape, or using this information in trigonometry or Pythagoras problems.
Proving two triangles are congruent.

COMMON MISCONCEPTIONS

Students commonly use the same scale factor for length, area and volume.
Students sometimes confuse the SAS condition with “an angle and 2 sides” (ASS).

NOTES

Encourage students to model (or consider) what happens to the area when a 1 cm square is enlarged by a scale factor of 3
Ensure that examples involving given volumes are used, requiring the cube root being calculated to find the length scale factor.
Ensure that students are exposed to a range of examples of congruent triangles using all 4 conditions.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 12, Q 17

29. Vectors

Teaching time
5-7 hours

OBJECTIVES

8 A	Scalar and vector quantities
8 B	understand and use vector notation including column vectors
8 C	Representation of a vector by a directed line segment
8 D	Parallel vectors, unit vectors and position vectors
8 E	Sum and difference of two vectors
8 F	Modulus (magnitude) of a vector
8 G	Multiplication of a vector by a scalar
8 H	find the resultant of two or more vectors
8 I	apply vector methods for simple geometrical proofs

POSSIBLE SUCCESS CRITERIA

Add and subtract vectors algebraically and use column vectors.
Solve geometric problems and produce proofs.
Find the magnitude of a vector.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

“Show that”-type questions are an ideal opportunity for students to provide a clear logical chain of reasoning, providing links with other areas of mathematics, in particular algebra.
Find the area of a parallelogram defined by given vectors.

COMMON MISCONCEPTIONS

Students find it difficult to understand that parallel vectors are equal as they are in different locations in the plane.

NOTES

Students find manipulation of column vectors relatively easy compared to pictorial and algebraic manipulation methods – encourage them to draw any vectors they calculate on the picture.
Geometry of a hexagon provides a good source of parallel, reverse and multiples of vectors.
Remind students to underline vectors or use an arrow above them, or they will be regarded as just lengths.
Extend geometric proofs by showing that the medians of a triangle intersect at a single point.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 7

30. Graphical representation of data	Teaching time 2-4 hours

OBJECTIVES

10 A	Graphical representation of numerical data e.g. bar charts, pie charts and histograms but not Cumulative frequency graphs

POSSIBLE SUCCESS CRITERIA

Calculate the size of angles in a pie chart given the data in a table.
Construct histograms from frequency tables.
Compare two data sets and justify their comparisons based on measures extracted from their diagrams, where appropriate, in terms of the context of the data.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Interpret two or more data sets from their pie charts or histograms and relate the key measures in the context of the data.

COMMON MISCONCEPTIONS

Labelling axes incorrectly in terms of the scales, and also using 'Frequency' instead of 'Frequency Density' .
Histograms are often not well understood with the height used for frequency rather than the area.

NOTES

Ensure that axes are clearly labelled.

EXAMPLE QUESTIONS FROM SAMs: none

31. Statistical measures

Teaching time
2-4 hours

OBJECTIVES

10 B	Determination of the mean, median, mode and range for a discrete data set
10 C	Calculation of an estimate of the mean of a larger number of quantities given in grouped frequencies
10 D	Determination of a modal class and the class containing the median for grouped data

POSSIBLE SUCCESS CRITERIA

Be able to state the median, mode, mean and range from a small data set.
Estimate the mean from a grouped frequency table.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to provide reasons for choosing to use a specific average to support a point of view.
Given the mean, median and mode of five positive whole numbers, can you find the numbers?
Students should be able to provide a correct solution as a counterargument to statements involving the "averages", e.g. Susan states that the median is 15, she is wrong. Explain why.
Find an estimate for the mean or the median from a histogram.

COMMON MISCONCEPTIONS

Students often forget the difference between continuous and discrete data.
Often the $\sum(m \times f)$ is divided by the number of classes rather than $\sum f$ when estimating the mean.

NOTES

Encourage students to cross out the midpoints of each group once they have used these numbers to in $m \times f$. This helps students to avoid summing m instead of f.
Remind students how to find the midpoint of two numbers.
Emphasise that continuous data is measured, i.e. length, weight, and discrete data can be counted, i.e. number of shoes.

EXAMPLE QUESTIONS FROM SAMs: 01 Q 23

32. Probability

Teaching time
5-7 hours

OBJECTIVES

10 E	understand the language and basic concepts of probability e.g. probability scale, sample spaces, relative frequency, probabilities and complements
10 F	Use of addition rule for two or more mutually exclusive events
10 G	Use of product rule for two or more independent events
10 H	Determination of the probability of two or more independent events including the use tree diagrams
10 I	Using simple conditional probability for combined events
10 J	Finding very simple conditional probability
10 K	understand and use the term 'expected frequency'

POSSIBLE SUCCESS CRITERIA

If the probability of outcomes of all values in the sample space are x, $2x$, $4x$, $3x$, calculate x.
Draw a Venn diagram of students studying French, German or both, and then calculate the probability that a student studies French given that they also study German.
Use a tree diagram to find the probability of a combined event.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to justify the probability of events happening or not happening in real-life and abstract contexts.
Questions may involve setting up and solving algebraic equations.

COMMON MISCONCEPTIONS

Probability without replacement is best illustrated visually and by initially working out probability 'with' replacement.
Not using fractions or decimals when working with probability trees.

NOTES

Encourage students to work 'across' the branches, working out the probability of each successive event. The probability of the combinations of outcomes should = 1
If a question says, for example, that 'two counters are taken from a bag' then, by implication, this is a non-replacement probability question.
Ensure students practise identifying simple conditional probabilities, e.g. "given that the student studies French find the probability that they also study German" compared with "find the probability that the student studies French and German".

EXAMPLE QUESTIONS FROM SAMs: 01 Q 9 and 02 Q 9

Transferable skills

<u>The need for transferable skills</u>

In recent years, higher education institutions and employers have consistently flagged the need for students to develop a range of transferable skills to enable them to respond with confidence to the demands of undergraduate study and the world of work.

The Organisation for Economic Co-operation and Development (OECD) defines skills, or competencies, as ‘the bundle of knowledge, attributes and capacities that can be learned and that enable individuals to successfully and consistently perform an activity or task and can be built upon and extended through learning.’ To support the design of our qualifications, the Pearson Research Team selected and evaluated seven global 21st-century skills frameworks. Following on from this process, we identified the National Research Council’s (NRC) framework as the most evidence-based and robust skills framework, and have used this as a basis for our adapted skills framework. The framework includes cognitive, intrapersonal skills and interpersonal skills.

The skills have been interpreted for this specification to ensure they are appropriate for the subject. All of the skills listed are evident or accessible in the teaching, learning and/or assessment of the qualification. Some skills are directly assessed.

The following table will support you in identifying these skills and developing these skills in students.

NRC framework skill	Skill interpretation in this subject	Where the skill is covered in content	Where the skill is explicitly assessed in examination	Opportunity for the skill to be learned through teaching and delivery
Cognitive skills				
Cognitive Processes and Strategies				
Critical thinking	Using **many** different pieces of mathematical information (sometimes seemingly unrelated) and synthesising this information to arrive at a solution to a mathematics-based problem.	e.g. 3I (quadratic and linear simultaneous equations) 9B (2D and 3D Trig)	Many of the longer questions on paper 2 address this e.g. Qu 7 (9B), Qu 8 (section 4). Qu 5 (3I)	Yes
Problem solving	Translating problems in mathematical or non-mathematical contexts into a	See note in 1K but examples occur over a range of topics	e.g. Paper 2 Qu 1 (section 1)	yes

	process or a series of mathematical processes and solve them.		Paper 1 Qu 21 (sections 1 & 3)	
Analysis	Examining and understanding different elements of a mathematical context or different mathematical processes.	e.g. Curves, sketching and using calculus (4 J, K, L, M, N) Links to solutions of equations (3G)	e.g. Paper 2 Qu 8 Paper 1 Qu 27 (Quadratics and inequalities) Qu 25 (section 4 N)	yes
Reasoning	Making abstract deductions and draw conclusions from mathematical information.	Examples in Geometry involving congruence (6 I, J), similarity (6 H) and circle theorems (6K, L). Also in vectors (8I) and matrices (5F)	e.g. Paper 1 Qu 17 (6I, J) Qu 24 (6K) Paper 2 Qu 11(c and d) (6H) Qu 10 (f and g) (5F)	
Interpretation	Analysing mathematical information and understanding the meaning of that information, for example interpreting straight line conversion graphs.	Examples in many areas e.g. 1G, 4N and Venn diagrams and probability (sections 2 and 10)	Paper 2 Qu 6 (4N) Qu 3 (Sections 2 and 10) Paper 1 Qu 25 (4N)	
Decision Making	Selecting a mathematical process from a series of mathematical processes to solve a problem.	Examples in trigonometry (9B) and Pythagoras' theorem (6F)	e.g. Paper 1 Qu 28 (9B) Paper 2 Qu 7 (9B & 6F)	e.g. Use of discussion in whole class contexts or in small groups.
Adaptive learning	Adapting a mathematical strategy to solve a context based mathematical problem.	Examples in percentages and proportions (1 H) and probability (section 10)	e.g. Paper 2 Qu 1 (1H) Qu 9 (section 10) Paper 1 Qu 14 (section 10)	
Executive function	Planning how to solve a problem, carrying out the plan and reviewing the outcome.	Many longer, unstructured questions	e.g. Paper 1 Qu 17 (6I, Paper 2 Qu 11(d) Qu 5 (3I)	
Creativity				
Creativity	Using own learning to apply mathematical processes and link these together to prove and validate mathematical concepts Uses a different, unexpected mathematical process to arrive at an answer.	We use "Show that" style of questions where candidates have to give something approaching a proof.	e.g. Paper 1 Qu 17 (6J) Qu 21 (number and algebra) Qu 26 (factor theorem 3D) Paper 2 Qu 4 (4M)	Yes May be evidenced in homework tasks

		Sometimes we use "prove" in the context of congruent triangles (6J)	Qu 11b (Vectors 8I)	
Innovation	Using a novel strategy to solve a previously unseen mathematical problem.	There is scope here in the area of turning points on curves (4M)	Hard to explicitly assess but candidates may produce solutions not on mark scheme. e.g. to find the *x*-coordinate of the minimum on $y = 3x^2 - 9x + 5$ the candidate uses ideas of symmetry and the mid-point of the roots. They may then use a knowledge that the sum of the roots is $-\frac{b}{a}$ to write down the answer as $\frac{1}{2} \times \frac{9}{3} = \frac{3}{2}$ rather than using calculus.	Yes See example.

NRC framework skill	Skill interpretation in this subject	Where the skill is covered in content	Where the skill is explicitly assessed in examination	Opportunity for the skill to be learned through teaching and delivery
Intrapersonal skills				
Intellectual openness				
Adaptability	Ability to select and apply knowledge and understanding of mathematical processes (that which is not prompted or provided) to unseen mathematical problems.	Many questions would assess this.	Yes Any question where we do not specify the method to use e.g. Paper 1 Qus 10 (1D), 12	

			(6G), 15 (4H, I), 18 (3H), 20(6K, L) Paper 2 Qus 2, 5 (Algebra)	
Personal and social responsibility	Using mathematical knowledge and skills to solve a problem for which one is accountable.	Section 1K has a note all about applying number in everyday use	e.g. Paper 2 Qu 1 (1H)	Yes e.g. students monitoring their allowance
Continuous learning	Planning and reflecting on own learning-setting goals and meeting them regularly			Yes Students identify areas where they need extra help or practice.
Intellectual interest and curiosity	Identifying a problem under own initiative, planning a solution and carrying this out.	e.g. the topic of sequences lends itself to this (3B and 3L)	e.g. Paper 1 Qu 5 <u>or</u> could give sequence 2, 8, 18, 32… and ask for the next two terms	Yes Student goes on to try and find a formula for the *n*th term (= $2n^2$) Not on specification but a simple question student could ask and explore.
Initiative	Using mathematical knowledge, independently (without guided learning), to further own understanding.			Yes Reading magazines such as “Plus” published by The Mathematical Association.
Self-direction	Planning and carrying out mathematical-based problem-solving under own direction.			Yes
Responsibility	Taking responsibility for any errors or omissions in own work and creating a plan to improve.	Section 1I covers rounding. Candidates can be encouraged to round to 1sf before evaluating an answer on a calculator.	e.g. Candidate may estimate answer as $\frac{50\times3}{10}$ before carrying out calculation of $\frac{51.2\times 2.96}{3.1+7.4}$ on a calculator.	Yes Teaching style can encourage candidates to ask if an answer is “reasonable” or estimate.
Perseverance	Actively seeking new ways to continue and improve own learning despite setbacks.			Yes
Productivity	Using mathematical strategies and problem solving skills fluently (?)	Some of the longer questions that require several steps would assess this.		Yes

Self-regulation (metacognition, forethought, reflection)	Developing and refining a strategy over time for solving a problem, reflecting on the success or otherwise of the strategy			Yes
Ethics	Producing output with a specific moral purpose for which one is accountable.			Yes
Integrity	Taking ownership for own work and willingly responds to questions and challenges.			Yes
Positive Core Self Evaluation				
Self-monitoring/self-evaluation/self-reinforcement	Planning and reviewing own work as a matter of habit.			Yes

NRC framework skill	**Skill interpretation in this subject**	**Where the skill is covered in content**	**Where the skill is explicitly assessed in examination**	**Opportunity for the skill to be learned through teaching and delivery**
Interpersonal skills				
Teamwork and collaboration				
Communication	Able to communicate a mathematical process or technique (verbally or written) to peers and teachers and answer questions from others.			Yes e.g. in group discussion
Collaboration	Carrying out a peer review to provide supportive feedback to another.			Yes
Teamwork	Working with other students in a maths-based problem solving exercise.			Yes
Co-operation	Sharing own resources and own learning techniques with other students.			Yes
Interpersonal skills	Using verbal and non-verbal communication skills in a dialogue about mathematics.			Yes
Leadership				
Leadership	Leading others in a group activity to effectively solve a mathematical problem			Yes

Responsibility	Taking responsibility for the outcomes of a team exercise even if one is not solely responsible for the output.			Yes
Assertive communication	Chairing a debate, allowing representations and directing the conversation to a conclusion.			Yes
Self-presentation	Presenting a mathematical problem to an audience to seek solutions.			Yes

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